1. The roof of a rectangular floor ($16m\ x\ 12m$) base structure can be modelled by

$$z(x,y) = 4 + \frac{y}{25} - \frac{x^2}{50}$$

(x,y) is the coordinate of the floor with $(0,0)$ as the coordinate of the centre of the floor, i.e. $x = -8$ to 8 and $y = -6$ to 6.

Use the online mathematical tools Wolfram Alpha to compute below (a), (b), (d) and (e).

Use Matlab (integral2 function) to compute below (f) and (g).

Show and explain the working steps, particularly on how those upper and lower limits are obtained for (d) and (e), together with the screen captures in your solutions.

a) Use Wolfram Alpha to show the 3D graph of the roof (i.e. function z) above the floor region $-8 \le x \le 8$ and $-6 \le y \le 6$. (4 marks)

b) Use Wolfram Alpha to calculate the volume of the structure by using both double integrals.

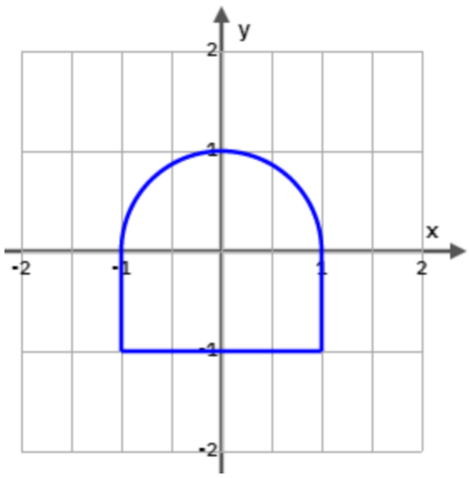
$$\int_a^b \left[\int_c^d z(x,y)\, dx \right] dy = \int_a^b \int_c^d z(x,y)\, dx\, dy$$ (4 marks)

$$\int_e^f \left[\int_g^h z(x,y)\, dy \right] dx = \int_e^f \int_g^h z(x,y)\, dy\, dx$$ (4 marks)

c) Not using any mathematical tools, manually compute the volume of the structure by using below double integrals.

$$\int_a^b \left[\int_c^d z(x,y)\, dx \right] dy = \int_a^b \int_c^d z(x,y)\, dx\, dy$$ (14 marks)

d) There is a pillar as shown below at the centre of the structure from the floor to the roof. Find the volume of the pillar by using below double integrals via Wolfram Alpha. Show and explain the working steps, particularly on how those upper and lower limits are obtained.



$$\int_{a}^{b} \int_{h(y)}^{H(y)} z(x, y)\, dx\, dy$$

(32 marks)

e) By using the reversed order double integrals to find the volume of the pillar in (d) via Wolfram Alpha. Show and explain the working steps, particularly on how those upper and lower limits are obtained.

$$\int_{c}^{d} \int_{g(x)}^{G(x)} z(x, y)\, dy\, dx$$

(18 marks)

f) Using Matlab (integral2 function) to find the volume of the pillar in (d) by using below double integrals. Use those limits that you have found in (d).

$$\int_{a}^{b} \int_{h(y)}^{H(y)} z(x, y)\, dx\, dy$$

(12 marks)

g) Using Matlab (integral2 function) to find the volume of the pillar in (d) by using the reversed order double integrals. Use those limits that you have found in (e).

$$\int_{c}^{d} \int_{g(x)}^{G(x)} z(x, y)\, dy\, dx$$

(6 marks)

h) By using the basic geometric method to estimate the volume of the pillar.

(6 marks)