York University

Faculty of Science

Math 1510
April 2019

Practice Examination

NAME (print legibly): _____   _____
(Family Name Here)   (First Name Here)

SIGNATURE: _____

STUDENT NUMBER: _____

SECTION: _____ (If you don't know your section, put the time of your class or your instructor's name).

Instructions:

1. Answer all 20 questions.

2. The exam is 3 hours.

3. No calculators, cellphones, or computers.

4. One handwritten 8.5" by 11" one-sided formula sheet is permitted. No other aids.

5. Show your work.

6. Use backs of pages as scrap paper.

Q	Marks	Q	Marks	Q	Marks
1		9		17	
2		10		18	
3		11		19	
4		12		20	
5		13			
6		14			
7		15			
8		16			
				Total	

Question 1 [6 marks]

Graph $f(x) = (x-2)^3$.

Is it symmetric with respect to the x-axis?

Is it symmetric with respect to the y-axis?

Is it symmetric with respect to the origin?

Is it an odd function?

Show $-f(-x)$ with a dotted line on the same graph.

Question 2 [6 marks]

Suppose $f(x) = \frac{x-1}{x^2+x}$ and $g(x) = \frac{x^3-x}{x^2+1}$.

What is $(f+g)(t-1)$?

What is $(fg)(a+b)$?

What is $(f \circ g)(z)$?

What is $(f \circ f)(2a)$?

You need not simplify your answers.

Question 3 [6 marks] Suppose $P(x) = 6x^5 - 5x^4 - 12x^3 - 6x^2 - 10x + 29$ and $D(x) = x - 2$.
Use long division to divide $P(x)$ by $D(x)$.
Use synthetic division to divide $P(x)$ by $D(x)$.
Identify $Q(x)$ and $R(x)$ and check that $P(x) = D(x)Q(x) + R(x)$.
Use the remainder theorem to calculate $P(2)$. Check your answer by substituting 2 into $P(x)$.

Question 4 [5 marks]

Graph $y = (x^4 - 2x^3 + x^2)(2-x)^3(x-4)^2$.

Indicate all x-intercepts and y-intercepts.

Clearly show what happens to the function at the far left and far right of the graph.

Indicate whether the graph crosses the x-axis at each x-intercept.

Question 5 [6 marks]

Suppose

$$g(x) = \frac{(2x-1)^3(x+1)^3}{(x-2)(2x-3)^2(x-4)^3}$$

What is the domain of g?

Identify all horizontal asymptotes and vertical asymptotes.

Describe what happens near each vertical asymptote.

Sketch a graph of g.

Question 6 [5 marks]

Suppose $h(x) = \frac{ax-2}{bx+4}$ where a, b are constants.

Calculate $h^{-1}(x)$ (your answer should look like $h^{-1}(x) = \ldots$.

What is the domain of h^{-1}?

Verify that $h(h^{-1}(x)) = x$.

Question 7 [5 marks]

A bar of tungsten is heated to $3400°C$ and placed in a cooling rack at 3 PM on April 18, 2019. The cooling rack is kept at $100°C$ Suppose the temperature T of the centre of the tungsten bar is given by

$$T(t) = A + Be^{-0.042t}$$

where t is the number of minutes after noon on April 18, 2019.

Eventually the tungsten will cool to $100°C$. So what is A?

Find B.

What is the temperature of the iron bar, after it has cooled for one hour?

(Your answers can have e in them, since you don't have a calculator).

Question 8 [4 marks]

Write $a^3 = \log_{e^2}(4 - x^4)$ in exponential form.

What is the domain of $f(x) = \log_{e^2}(4 - x^4)$?

If $x = \sqrt{\sqrt{3}}$, then what is $f(x)$? (Your answer should be an integer).

Question 9 [5 marks]
Part 1: Fully expand the expression

$$\ln\left(\frac{(2x)^3(x+1)^2}{(x+2)^3(x+3)^5}\right)$$

Your answer should have no exponents and can only have ln of $x, x+1, x+2, x+3$ or a constant.
Part 2: Evaluate the expression $\log_6(30) - \log_6(27) - \log_6(40)$. (Hint: Write it as a single logarithm; your answer should be an integer).
Part 3: Write $\log_{e^3}(4^{-x})$ in terms of ln by using the change of base formula (Your answer should have no exponents, but it can have ln).

Question 10 [4 marks]
The decibel level dB of a sound with intensity I is given by

$$dB(I) = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where I_0 is the intensity of sound that is barely audible to the human ear.
The intensity of a jet engine is $(1.58 \times 10^{15}) \cdot I_0$. What is the decibel level? Your answer can use $\log_{10}$.
How many times greater is the intensity of a sound that measures 123 dB, than one that measures 94 dB? Your answer should be a power of 10.
Give your answers in sentences.

Question 11 [4 marks]

Solve

$$\frac{10^x - 10^{-x}}{10^x + 10^{-x}} = \frac{1}{3}$$

Solve

$$\ln\left((2x^2 + 3x - 4)^8\right) = 16 \ln x$$

Question 12 [4 marks]

A pulley A with a radius of 5 cm uses a belt to drive a pulley B with a radius of 11 cm. Pulley A turns at 6 revolutions per minute.

How many seconds does it take for pulley B to revolve once?

How many seconds does it take for pulley A to turn by one degree?

If pulley B turns $420°$, then how many radians does pulley A turn ?

If pulley B turns 2 radians, then find the distance that the belt around pulley A moves.

Question 13 [6 marks]
The deer population is a nature reserve is given by the function

$$P(t) = \frac{P_0}{1 - \frac{1}{3}e^{-bt}}$$

for some constants P, a and b where t is the time in years since January 1, 2019.
If the population on January 1, 2023 is twice the population on January 1, 2019, then find b. Your answer can have ln in it.

Question 14 [5 marks]
Suppose θ is an angle between π and $3\pi/2$ and $\csc(\theta) = 2$.
Find $\cos(\theta)$.
Find $\tan(\theta)$.
Find θ (in radians).

Continues...

Question 15 [4 marks]

Two people A and B are looking at the top of a radio tower. To person A, the angle of elevation is $30°$ and to person B, the angle of elevation is $45°$. Person B is 300 m closer to the tower than person A. How high is the radio tower? Give your answer in a sentence.

Question 16 [6 marks]

In which quadrant are each of the angles (in radians): $11\pi/3$ and $17\pi/6$ and $-44\pi/3$?

Will $\cot(x)$ be positive or negative for each of these three angles?

What is the reference angle of each of these three angles?

Calculate the cotangent of each of the three angles.

Continues...

Question 17 [6 marks]

The hours of daylight hours in Toronto on the nth day of the year is given by:

$$D(n) = 12 + 3.25 \sin\left(\frac{2\pi(t - 80)}{365}\right)$$

How many daylight hours are there on January 26?

How long is the longest day of the year?

There are 12 daylight hours on the 80th day of the year. When does this next happen again?

What is the period of $D(n)$?

Sketch the graph of $D(n)$.

Question 18 [4 marks]

$$a_n = (-1)^n \frac{n^2 + 1}{n + 1}$$

Calculate a_{10}.

Write $a_{10} + a_{11} + a_{12} + \ldots + a_{45}$ using summation notation (without using the letter a).

Write $(-79) + (-75) + (-71) + \ldots + 197 + 201$ using summation notation.

Question 19 [4 marks]

Explain why $-79, -75, -71, -67, \ldots$ is an arithmetic sequence.

Find the general term a_n.

Find the nth partial sum S_n of this arithmetic sequence.

Add up $(-79) + (-75) + (-71) + \ldots + 197 + 201$.

Question 20 [5 marks]

Explain why $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$ is a geometric sequence.

Find the general term a_n of this sequence.

Compute $\sum\limits_{n=1}^{100} a_n$.

Compute $\sum\limits_{n=1}^{\infty} a_n$.

End of Exam.