1. $xu_x + u_y = x$

$u(0) = 0 = u_0\left(x_0(s), y_0(s)\right) = u_0(s, 0) = 0, \quad x > 0$

$\Gamma = \{(s, 0) | s > 0\}$

$\frac{dx}{d\tau} = x, \qquad x(0) = x_0(s) = s$

$x = se^{\tau}$

$\frac{dy}{d\tau} = 1, \qquad y(0) = y_0(s) = 0$

$y = \tau$

$\frac{du}{d\tau} = x, \qquad u(0) = u_0(s) = 0$

$\frac{\partial u}{\partial x} \times \frac{dx}{d\tau} = \frac{du}{dx} = x$

$1\ du = x\ dx$

$u = \frac{x^2}{2}$

$u(s, \tau) = \frac{(se^t)^2}{2}$

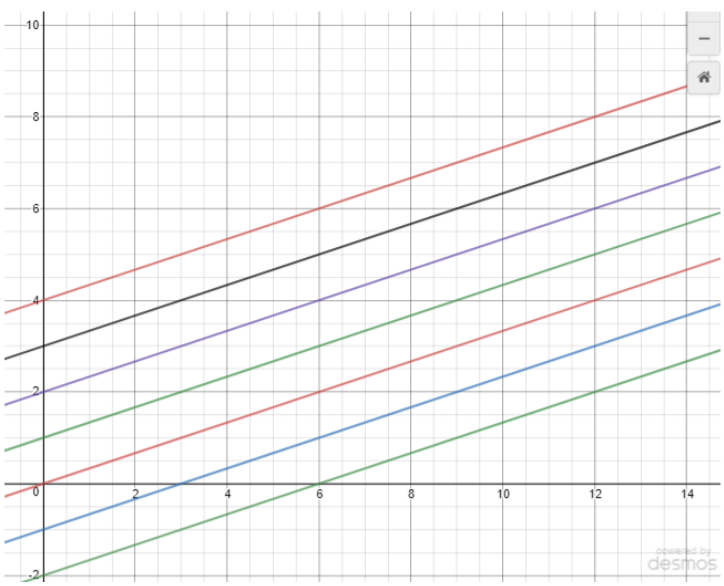
$u(x, y) = \frac{(xe^{-y}e^{y})^2}{2} = u(x, u) = \frac{x^2}{2}$

$x = se^{y}$

$s = xe^{-y}$

$y = ln\left(\frac{x}{s}\right)$

$Slope = \frac{1}{3}$ characteristics



$u(x, y) =$

2. $xu_{xx} - 4yu_{xy} = 0$

$x_0(s) = c_1(s)$
$y_0(s) = c_2(s)$
$u(x_0(s), y_0(s)) = c_3(s)$

$\frac{dx}{d\tau} = x, \qquad x(0,s) = c_1(s)$

$\frac{dt}{d\tau} = -4y, \qquad y(0,s) = c_2(s)$

$\frac{du}{d\tau} = 0, \qquad u(0) = c_3(s)$

3. $u'' - u = f(x)$

$$u(0) = u(1)$$
$$u'(0) = u'(1)$$

1.

(a) Find the solution to the Cauchy problem, for $x > 0$,

$$xu_x + u_y = x,$$

with $u(x, 0) = 0$, **AND** sketch the characteristics.

(b) Check your solution is correct.

(c) Find the form of the general solution. Check your answer.

(d) If instead, $u(x, \ln x) = g(x)$. Show that the solution may not exist unless g satisfies has a certain form. Find the solution if g does have this form. Show the Fundamental Existence Theorem does not apply.

(e) Show the solution in Part (*d*) is not unique. You may find the general solution in (*c*) useful.

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2. Find the general solution to

$$xu_{xx} - 4yu_{xy} = 0.$$

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3. Consider the boundary-value problem

$$u'' - u = f(x),$$
$$u(0) = u(1)$$
$$u'(0) = u'(1).$$

(a) Construct a Green's function
Hint: To reduce the number of unknowns look for the Green's function in the form

$$G(x, x') = \begin{cases} c_1 e^{x'} + c_2 e^{-x'} & x' < x \\ c_3 e^{x'-1} + c_4 e^{-(x'-1)} & x' > x \end{cases}$$

Also, because this problem is self adjoint (the adjoint problem is the same as the original), you will find $G(x, x') = G(x', x)$ whe you are done.

(b) Find the solution $u(x)$ in term of the Green's function. It will only involve an integral because the boundary conditions are homogeneous.

(c) Check your answer using Leibniz's rule (Theorem 5.2 in the notes).